# GCSE MATHEMATICS

S21-C300

## With Calculator Assessment Resource P

Higher Tier

**Formula list**

*Area and volume formulae*

Where $r$ is the radius of the sphere or cone, $l$ is the slant height of a cone and $h$ is the perpendicular height of a cone:

Curved surface area of a cone = $\pi rl$

Surface area of a sphere = $4\pi r^2$

Volume of a sphere = $\frac{4}{3}\pi r^3$

Volume of a cone = $\frac{1}{3}\pi r^2 h$

*Kinematics formulae*

Where $a$ is constant acceleration, $u$ is initial velocity, $v$ is final velocity, $s$ is displacement from the position when $t = 0$ and $t$ is time taken:

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

**1.** *(a)* Emma buys a car for £6500.
She later sells it for £5720.

Calculate her percentage loss. [2]

$\frac{5720}{6500} \times 100 = 88\%$.

$100\% - 88\% = 12\%$ percentage loss

*(b)* Emma buys another car for £8495.
Its value decreases by 16% each year.

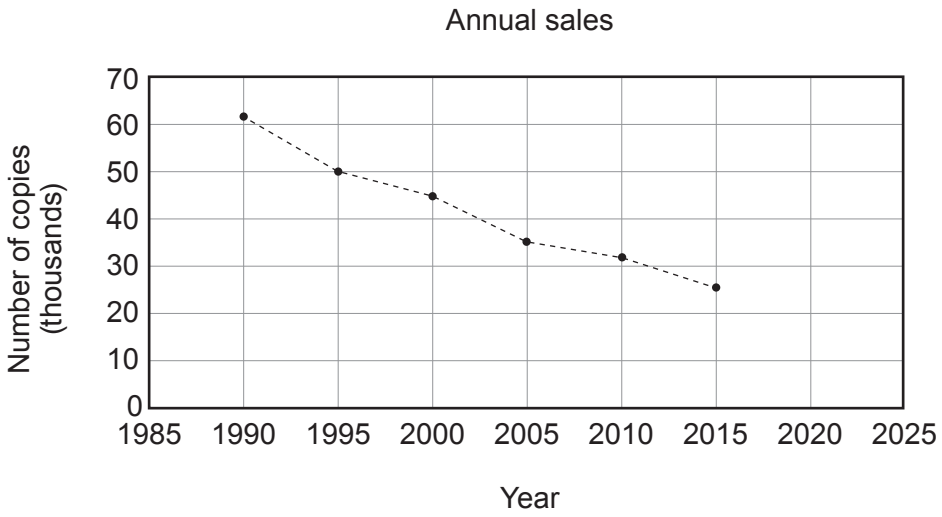
What is the car's value after 11 years? [3]

$8495(1 - 0.16)^{11}$

$= 1248.060188$

$\approx £1248.06$

**2.** The graph shows the number of copies of a local newspaper sold over a 25-year period.




*(a)* Eva uses the graph to predict that about 10 thousand newspapers will be sold in 2025.

Explain why her prediction may not be reliable. [1]

as 2025 will be extrapolated data so may not be reliable

*(b)* The ratio of adults who read news online to those who do not is 16 : 9.
The adult population of the UK is about 52 000 000.

Calculate an estimate of the number of adults in the UK who read news online. [2]

16 + 9 = 25

$\frac{52\,000\,000}{25} = 2\,080\,000$

16 x 2 080 000 = 33 280 000 adults in the UK

**3.** *(a)* 7476 football supporters watched the first match of the season.

The ratio of men : women : children was 10 : 8 : 3.

Show that 712 more men than women watched the match. [2]

$10 + 8 + 3 = 21$

$\frac{7476}{21} = 356$

$\therefore$ men $= 10 \times 356 = 3560$

women $= 8 \times 356 = 2848$

$3560 - 2848 = 712$, $\therefore$ 712 more men watched the match.

*(b)* At the second match of the season, the ratio of adults : children was 5 : 3.

At the third match, $\frac{2}{3}$ of the supporters were adults.

At which of these two matches was the proportion of adults higher?

You must show your working. [2]

Second match ☐ Third match ☑

2nd match : $\frac{5}{8} = \frac{15}{24}$

3rd match : $\frac{2}{3} = \frac{16}{24}$

$\frac{2}{3} > \frac{5}{8}$ $\therefore$ Third match

**4.** A full bottle containing 1 litre of cooking oil has mass 1270 g.
400 ml of cooking oil is used.

The bottle with the remaining cooking oil has mass 900 g.

Calculate the mass of the empty bottle. [4]

full bottle = 1270 g

0.6 litres left = 900 g

mass of empty bottle :

loss of mass = 1270 − 900 = 370 g

370 g = 400 ml

92.5 = 100 ml (ow)

∴ 1 litre of oil (excluding bottle) = 925 g

Mass of bottle = 1270 − 925

= 345 g

**5.** The mass of the planet Mercury is $3{\cdot}30 \times 10^{23}$ kg.
The volume of the planet Mercury is $6{\cdot}08 \times 10^{19}$ m$^3$.

Calculate the density of the planet Mercury in kg / m$^3$.
Give your answer to 3 significant figures. [3]

density = $\frac{\text{mass}}{\text{volume}} = \frac{3 \cdot 3 \times 10^{23}}{6.08 \times 10^{19}} = 5427.681579$

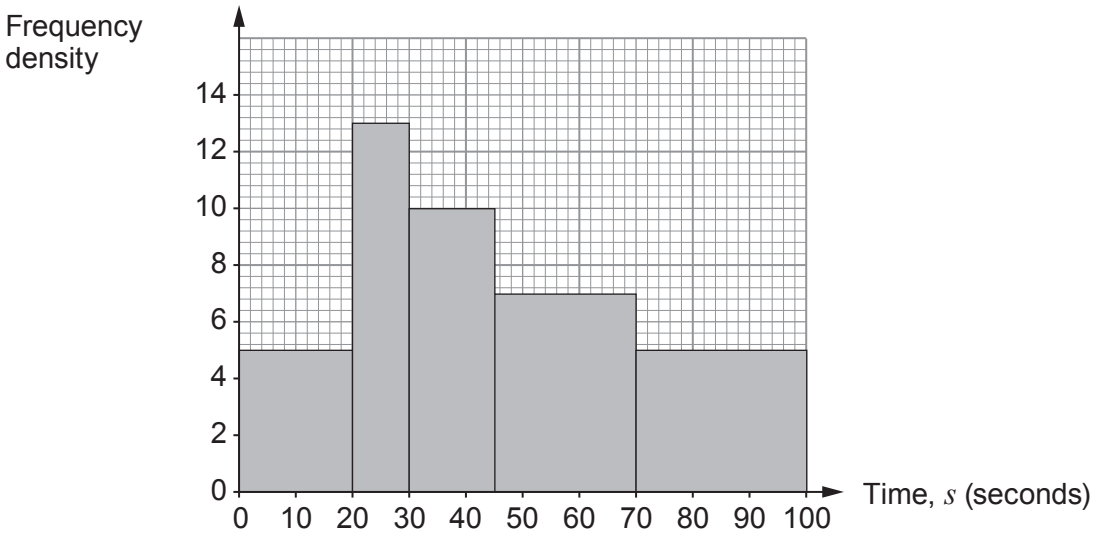
$\approx 5430$ (3sf)

Density = 5430 kg / m$^3$

**6.** *(a)* Freya records how long each of 40 people can hold their breath.
The results are shown in the table.

| Time, $s$ (seconds) | Frequency |
|---|---|
| $0 < s \leqslant 20$ | 5 |
| $20 < s \leqslant 30$ | 13 |
| $30 < s \leqslant 45$ | 10 |
| $45 < s \leqslant 70$ | 7 |
| $70 < s \leqslant 100$ | 5 |

Freya wants to draw a histogram for this data.
This is the graph she draws.



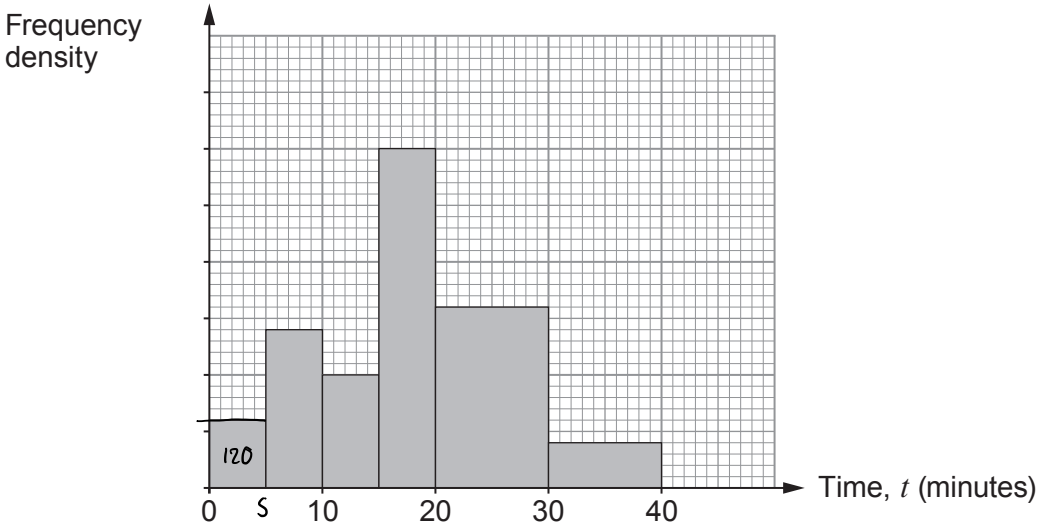

Has Freya drawn a histogram?

Yes ☑ No ☐

Give a reason for your answer. [1]

as she has used frequency density on her y-axis and class intervals on the x axis

*(b)* In one month, 2000 patients visited a doctors' surgery.
This histogram shows information about the length of time, $t$ minutes, these 2000 patients spent at the surgery.




The group $0 < t \leqslant 5$ represents 120 patients.

How many patients are represented by the group $30 < t \leqslant 40$? [2]

frequency density = $\frac{\text{frequency}}{\text{class width}}$

→ fd = $\frac{120}{5}$ = 24

24 ÷ 6 = 4 for each small square

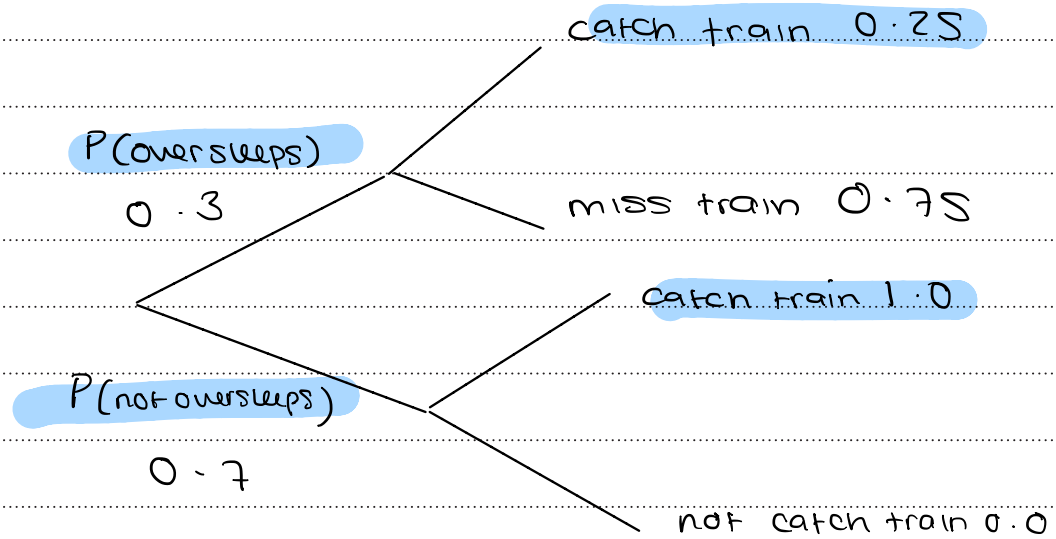
fd of $30 < t \leqslant 40$ = 4 x 4 = 16

16 x 10 = 160

**7.** *(a)* On any working day, the probability that Don oversleeps in the morning is 0·3.

When he oversleeps, the probability that he catches his train to work is 0·25.
When Don does not oversleep, he always catches his train.

Work out the probability that, on a randomly chosen working day, Don catches his train to work. [3]




$(0.3 \times 0.25) + (0.7 \times 1)$

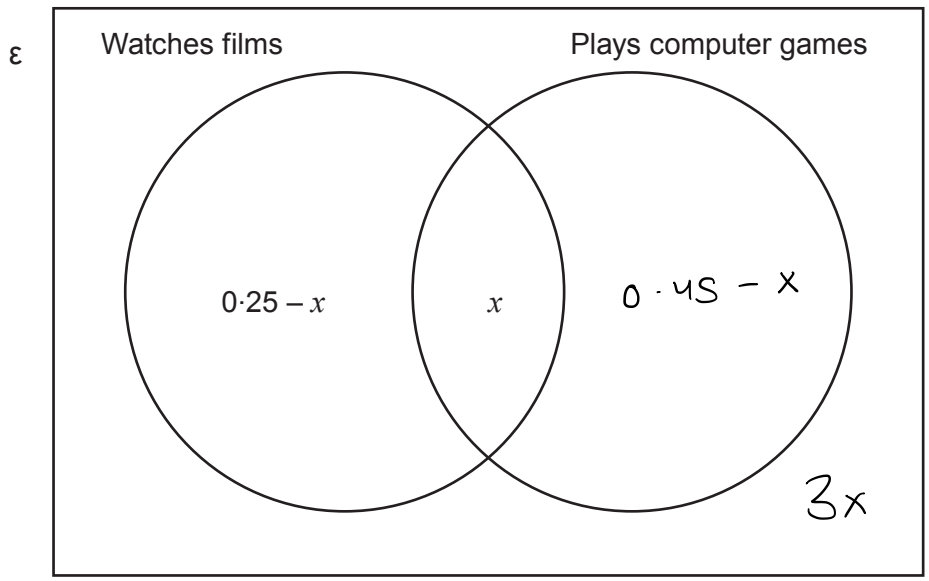
$= 0.075 + 0.7$

$= 0.775$

*(b)* Don sometimes spends his evenings watching films, playing computer games, or doing both.

On any evening the probability that Don:
- watches films is 0·25,
- plays computer games is 0·45,
- does neither is three times the probability that he does both.

(i) Complete the Venn diagram. [1]




(ii) Work out the probability that, on any randomly chosen evening, Don watches films **and** plays computer games. [2]

$0.25 - x + x + 0.45 - x + 3x = 1$

$0.7 + 2x = 1$

$2x = 0.3$

$x = 0.15$

$\therefore P(\text{both}) = 0.15$

(iii) On the evenings Don watches films, what is the probability that he also plays computer games? [2]

$P(\text{plays games} \mid \text{watches films}) = \frac{x}{0.25} = \frac{0.15}{0.25}$

$= 0.6$

**8.** When a ball is thrown upwards on the Moon, the maximum height, $h$ metres, it reaches is given by the formula $h = \frac{U^2}{2a}$.

In a particular case, $U$ = 4·2 and $a$ = 1·6, both correct to 2 significant figures.

Calculate the greatest possible value of $h$. [3]

In the particular instant

max $U = 4.35$

min $a = 1.55$

$$h = \frac{4.35^2}{2(1.55)} = 6.104032258 \approx 6.1\text{m}$$

**9.** *(a)* The number of voles, $V$, on an island $t$ years after the first voles are introduced is given by the formula

$$V = 135 \times 1{\cdot}06^t.$$

(i) How many voles were initially introduced? [1]

$V = 135 \times 1{\cdot}06^0$

$= 135$

(ii) What is the percentage increase in the number of voles 5 years after they were introduced? [2]

$V_5 = 135 \times 1{\cdot}06^5 = 180.660453$

or $\dfrac{180.66 - 135}{135} \times 100 = 33.82\%$

or $\dfrac{180.66\ldots}{135} - 1 = 0.338225576$

$0.338 \times 100 = 33.8\%$ increase

(iii) When the number of voles reaches 500, the population starts decreasing at a rate of 5% **per month**.

The formula $V = 500 \times k^T$ is now used to model the number of voles, $V$, where $T$ is the number of **years** after the population reached 500.

What value of $k$ should be used? [1]

0.05 per month $\rightarrow$ $0.05 \times 12 = 0.6$ per year

$1 - 0.6 = 0.4$ $\quad k = 0.4$

*(b)* A population of birds on the island has a constant growth rate, $p$%, per year.
There were initially 300 birds.
The population doubles in 20 years.

Calculate the value of $p$. [3]

$600 = 300\left(1 + \frac{p}{100}\right)^{20}$

$2 = \left(1 + \frac{p}{100}\right)^{20}$

$\sqrt[20]{2} = 1 + \frac{p}{100}$

$103.5264924 = 100 + p$

$3.5264923884 = p$

$p = 3.5\%$ (2sf)